PHYSICS 24 MIDTERM W18

(1) Before After

Use $c=1$, restore factors of c at the end

Remember, for photon $p = E/c$

Initial momentum W
Final Momentum p $\Longrightarrow$ $\underline{p = W}$

Initial energy $W+M$
Final energy $\sqrt{M'^2 + p^2}$

Conservation of energy

$$W+M = \sqrt{M'^2 + p^2} = \sqrt{M'^2 + W^2}$$

Square both sides

$$W^2 + M^2 + 2MW = M'^2 + W^2$$

$$M' = \sqrt{M^2 + 2MW}$$

Since W has units of energy, restore factors of c to get units right

$$\boxed{M' = \sqrt{M^2 + \frac{2MW}{c^2}}}$$

(2) From the point of view of earth two events

(a)

$\boxed{\beta = v/c}$

Length contraction $L' = \sqrt{1-\beta^2}\, L \quad L'' = \sqrt{1-4\beta^2}\, L$

Eqtn of motion for front of slow ship $x = L' + vt$

Eqtn of motion for back of fast ship $x = -L'' + 2vt$

At the coordinates (x,t) of the second event

$$L' + vt = -L'' + 2vt \Rightarrow t = \frac{L' + L''}{v} = \underline{\underline{\left[\sqrt{1-\beta^2} + \sqrt{1-4\beta^2}\right] L/v}}$$

And also $x = L' + vt = \left[2\sqrt{1-\beta^2} + \sqrt{1-4\beta^2}\right] L$

(b) Two ways of doing this. First way: coordinate transf

Let's go to the rest frame of the slow ship

$$t' = \gamma\left(t - \frac{vx}{c^2}\right) \quad \text{with } \gamma = \frac{1}{\sqrt{1-\beta^2}}$$

Substitute (x,t) from part (a)

$$t' = \frac{L}{v} + \frac{\sqrt{1-4\beta^2}}{\sqrt{1-\beta^2}}\frac{L}{v} - \frac{2vL}{c^2} - \frac{vL}{c^2}\frac{\sqrt{1-4\beta^2}}{\sqrt{1-\beta^2}} \qquad \text{Note } \frac{vL}{c^2} = \beta^2 \frac{L}{v}$$

$$t' = \frac{L}{v}\left[1 - 2\beta^2 + \frac{\sqrt{1-4\beta^2}}{\sqrt{1-\beta^2}} - \beta^2 \frac{\sqrt{1-4\beta^2}}{\sqrt{1-\beta^2}}\right]$$

$$t' = \frac{L}{v}\left[1 - 2\beta^2 + (1-\beta^2)\sqrt{\frac{1-4\beta^2}{1-\beta^2}}\right]$$

$$t' = \frac{L}{v}\left[1 - 2\beta^2 + \sqrt{(1-\beta^2)(1-4\beta^2)}\right]$$

$$\boxed{t' = \frac{L}{v}\left[1 - 2\beta^2 + \sqrt{1-5\beta^2+4\beta^4}\right]}$$

Nobody did it using this method on the test ☹

Second way of doing it

Find velocity of fast ship in rest frame of ~~2nd~~ slow ship. Call this velocity v' and work with $\beta' = v'/c$

$$\beta' = \frac{2\beta - \beta}{1-2\beta^2} = \frac{\beta}{1-2\beta^2}$$

The length of the fast ship in rest frame of slow ship will look contracted as

$$L''' = \sqrt{1-\beta'^2}\, L = \sqrt{1-\frac{\beta^2}{(1-2\beta)^2}}\, L = \frac{\sqrt{1+4\beta^4-5\beta^2}}{1-2\beta^2}$$

Then in this frame the fast ship will need to travel a distance $d = L + L'''$ in order to overtake the other ship. The fast ship is traveling at velocity $v' = \beta' c$

$\Rightarrow$ The time taken will be $t' = \dfrac{d}{\beta' c}$

$$t' = \frac{L + L'''}{\beta' c} = \frac{L}{c}\,\frac{1-2\beta^2}{\beta}\left[1 + \frac{\sqrt{1+4\beta^4-5\beta^2}}{1-2\beta^2}\right]$$

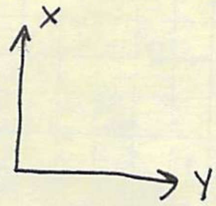
$$t' = \frac{L}{\beta c}\left[1-2\beta^2 + \sqrt{1+4\beta^4-5\beta^2}\right]$$

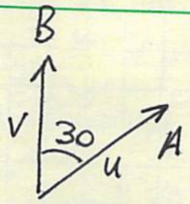
But $\beta c = v$

$$\boxed{t' = \frac{L}{v}\left[1-2\beta^2 + \sqrt{1-5\beta^2+4\beta^4}\right]}$$

And this is the method used by many people

(3)





$v = \frac{1}{2} c$

$u = \frac{1}{\sqrt{3}} c$

(4)

$$u_x = u \cos 30 = \frac{1}{\sqrt{3}} \frac{\sqrt{3}}{2} c = c/2$$

$$u_y = u \sin 30 = \frac{1}{\sqrt{3}} \frac{1}{2} c = \frac{c}{2\sqrt{3}}$$

Boost into rest frame of B $\qquad \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-\frac{1}{4}}} = \frac{2}{\sqrt{3}}$

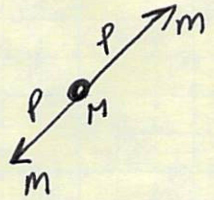
$$u_x' = \frac{u_x - v}{1 - \frac{v u_x}{c^2}} = 0 \quad \text{since } u_x = v = c/2$$

$$u_y' = \frac{u_y}{\gamma\left(1 - \frac{v u_x}{c^2}\right)} = \frac{\sqrt{3}}{2} \frac{c}{2\sqrt{3}} \frac{1}{1 - \frac{1}{2}\frac{1}{2}} = \frac{c}{4} \frac{4}{3} = \frac{c}{3}$$

Then $\quad u' = \sqrt{u_x'^2 + u_y'^2} \Rightarrow \boxed{u' = 1/3\, c}$

4

Use $c=1$, set the appropriate factors of c at the end



Cons. of energy $M = \sqrt{m^2+p^2} + \sqrt{m^2+p^2}$

$$M = 2\sqrt{m^2+p^2}$$

$$M^2 = 4m^2 + 4p^2$$

$$p^2 = \frac{M^2 - 4m^2}{4}$$

$$p = \frac{1}{2}\sqrt{M^2-4m^2}$$

Restore factors of c so that $[p] = [M][v]$

$$\boxed{p = \frac{c}{2}\sqrt{M^2-4m^2}}$$