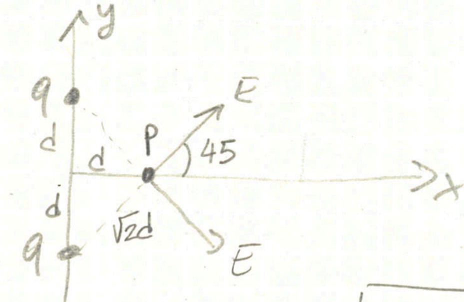
PHYSICS 24 MIDTERM

1 (a)



S

The y component cancels $\boxed{E_y = 0}$

The x components add

$E_x = 2E\cos 45 = \sqrt{2}E = \sqrt{2}\,\frac{1}{4\pi\varepsilon_0}\frac{q}{2d^2}$ $\boxed{E_x = \frac{1}{\sqrt{2}\,4\pi\varepsilon_0}\frac{q}{d^2}}$

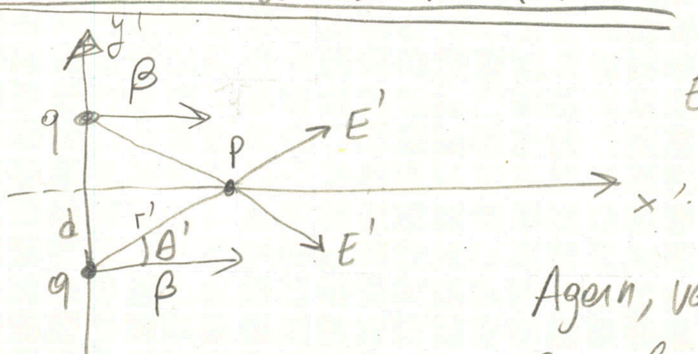
(b) The y-component is changed by a factor of γ

The x-component is unchanged

$\boxed{E'_y = 0}$ $\boxed{E'_x = \frac{1}{\sqrt{2}}\frac{1}{4\pi\varepsilon_0}\frac{q}{d^2}}$

This is using the laws for transformation of fields

Also, we can do it brute force



$$E' = \frac{q}{4\pi\varepsilon_0 r'^2}\frac{1-\beta^2}{(1-\beta^2\sin^2\theta')^{3/2}}$$

Again, vertical components cancel $\boxed{E'_y = 0}$

$$E'_x = 2E'\cos\theta'$$

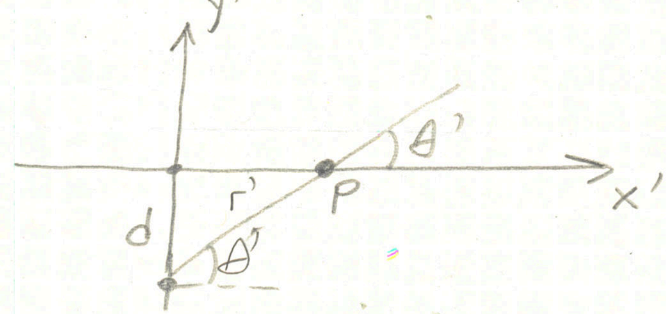
x'-components add

$$E' = \frac{1}{4\pi\varepsilon_0}\frac{q}{r'^2}\frac{1-\beta^2}{(1-\beta^2\sin^2\theta')^{3/2}}$$

Let's figure out the coordinates of P

$x = \gamma(x' - \beta t')$ at $t'=0$ $x = \gamma x'$ $\Rightarrow$ $x' = \frac{x}{\gamma} = \frac{d}{\gamma}$

And $y' = 0$



$\cos\theta' = \frac{x'}{r'} = \frac{d}{\gamma r'}$ $\sin\theta' = \frac{d}{r'}$

$$\text{Then } E'_x = 2E'\cos\theta' = \frac{2q}{4\pi\varepsilon_0 r'^2}\overbrace{\frac{d}{\gamma r'}}^{\cos\theta'}\frac{1-\beta^2}{(1-\beta^2\underbrace{d^2/r'^2}_{\sin^2\theta'})^{3/2}}$$

$$E'_x = \frac{2q}{4\pi\varepsilon_0}\frac{d}{r'^3}\frac{(1-\beta^2)^{3/2}}{\left(\frac{r'^2-\beta^2 d^2}{r'^2}\right)^{3/2}}$$

$$E'_x = \frac{2q}{4\pi\varepsilon_0}\frac{d(1-\beta^2)^{3/2}}{(r'^2-\beta^2 d^2)^{3/2}}$$

$$\text{But } r'^2 = d^2 + x'^2 = d^2 + \frac{d^2}{\gamma^2} = d^2 + d^2(1-\beta^2) = d^2(2-\beta^2)$$

$$E'_x = \frac{2q}{4\pi\varepsilon_0}\frac{d(1-\beta^2)^{3/2}}{[2d^2-\beta^2 d^2-\beta^2 d^2]^{3/2}} = \frac{2q}{4\pi\varepsilon_0}\frac{d(1-\beta^2)^{3/2}}{2\sqrt{2}\,d^3(1-\beta^2)^{3/2}}$$

$$\boxed{E'_x = \frac{1}{\sqrt{2}}\frac{1}{4\pi\varepsilon_0}\frac{q}{d^2}}$$

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2 We will use $c=1$

(a) By conservation of momentum both photons will have the same momentum, and therefore the same energy $\boxed{E = P = \frac{1}{2}M}$

since total energy $= M$

(b) S' is moving in the <u>negative x-direction</u> wrt S

Let's $v =$ this frame velocity and $\beta \equiv v/c$

This velocity (in magnitude) is also the velocity of the particle in S'.

$$P = \gamma M \beta \quad E = \gamma M \quad \Rightarrow \quad \beta = \frac{P}{E} \qquad \beta = \frac{P}{\sqrt{P^2+M^2}}$$

and, keeping track of the signs $\boxed{\beta = -\frac{P}{\sqrt{P^2+M^2}}}$

(c) For photon, in S frame, $P_x = 0 \quad P_y = \frac{M}{2} \quad E = \frac{M}{2}$

In S' frame

$$P_x' = \gamma(P_x + \beta E) = \gamma\beta\frac{M}{2} = \frac{1}{2}P$$

$$P_y' = P_y = \frac{M}{2}$$

$$\Rightarrow \tan\theta' = \frac{P_y'}{P_x'} = \frac{M/2}{P/2} \qquad \boxed{\tan\theta' = \frac{M}{P}}$$

$$\text{or} \quad \sin\theta' = \frac{P_y'}{\sqrt{P_x'^2 + P_y'^2}} = \frac{M/2}{\sqrt{\frac{1}{4}P^2 + \frac{1}{4}M^2}} \qquad \boxed{\sin\theta' = M/\sqrt{P^2+M^2}}$$

$$\boxed{\cos\theta' = P/\sqrt{P^2+M^2}}$$

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$\beta = 0.8 = 8/10 \qquad L = 3\ \text{Km}$

3 (a) $\gamma = \dfrac{1}{\sqrt{1-\beta^2}} = \dfrac{1}{\sqrt{1-64/100}} = \dfrac{10}{\sqrt{36}} = \dfrac{10}{6} = \dfrac{5}{3}$

The observer on the train feels the lorentz contraction of the distance between towers

$$\Delta t = \frac{L/\gamma}{v} = \frac{3}{5}\ \frac{3\ 10^3\,\text{m}}{0.8\ 3\ 10^8\,\text{m/sec}} = \underline{\underline{7.5\,\mu\text{sec}}}$$

(b) $\Delta t = \dfrac{L}{v} = \dfrac{3\ 10^3\,\text{m}}{0.8\ 3\ 10^8\,\text{m/sec}} = \underline{12.5\,\mu\text{sec}}$

4 (a) $v' = \dfrac{v+v}{1+\frac{vv}{c^2}} = \underline{\underline{\dfrac{2v}{1+v^2/c^2}}}$

(b) $v'' = \dfrac{v'+v}{1+\frac{vv'}{c^2}} = \left(\dfrac{2v}{1+v^2/c^2} + v\right) \Big/ \left(1 + \dfrac{v}{c^2}\dfrac{2v}{1+v^2/c^2}\right)$

$$v'' = \left(\frac{2v+v+v^3/c^2}{1+v^2/c^2}\right) \Big/ \left(1+\frac{2v^2}{c^2+v^2}\right) = \left(\frac{3v+v^3/c^2}{1+v^2/c^2}\right) \Big/ \left(\frac{c^2+v^2+2v^2}{c^2+v^2}\right)$$

$$v'' = \left(\frac{3v+v^3/c^2}{1+v^2/c^2}\right) \Big/ \left(\frac{1+3v^2/c^2}{1+v^2/c^2}\right)$$

$$\boxed{v'' = \frac{3v+v^3/c^2}{1+3v^2/c^2}}$$

Sanity check: for $v \ll c$ should reduce to $v'' = 3v$ ✓ yes!