Homework 7

1

7.21 emf $\mathcal{E} = -\dfrac{d\Phi}{dt}$

$N = 4000$
$A = \pi r^2$
$B_0 = 0.5\,G = 5\,10^{-5}\,T$
$\omega = 2\pi f$

$$\Phi = NAB_0 \cos(\omega t + \phi)$$

$$\mathcal{E} = N(\pi r^2) B_0 2\pi f \sin(\omega t + \phi)$$

$$\mathcal{E}_{MAX} = 2N\pi^2 r^2 f B_0 = 2 \cdot 4\,10^3 \cdot \pi^2 (12\,10^{-2})^2 30 \cdot 5\,10^{-5}\ V$$

$$\boxed{\mathcal{E}_{MAX} = 1.7V}$$

7.22 Consider circle of radius $R = 3\,cm$ centered on axis $\Phi = \pi R^2 B$

$B = B_0 \cos(\omega t + \phi)$ $B_0 = 4\,G = 4\,10^{-4}\,T$

$\omega = 2\pi f$ $f = 2.5\,10^6$

emf: $\mathcal{E} = -\dfrac{d\phi}{dt} = \underbrace{\omega \pi R^2 B_0}_{} \sin(\omega t + \phi)$

amplitude of emf $= \mathcal{E}_0$

$$\mathcal{E} = \oint \vec{E}\, d\vec{l}$$

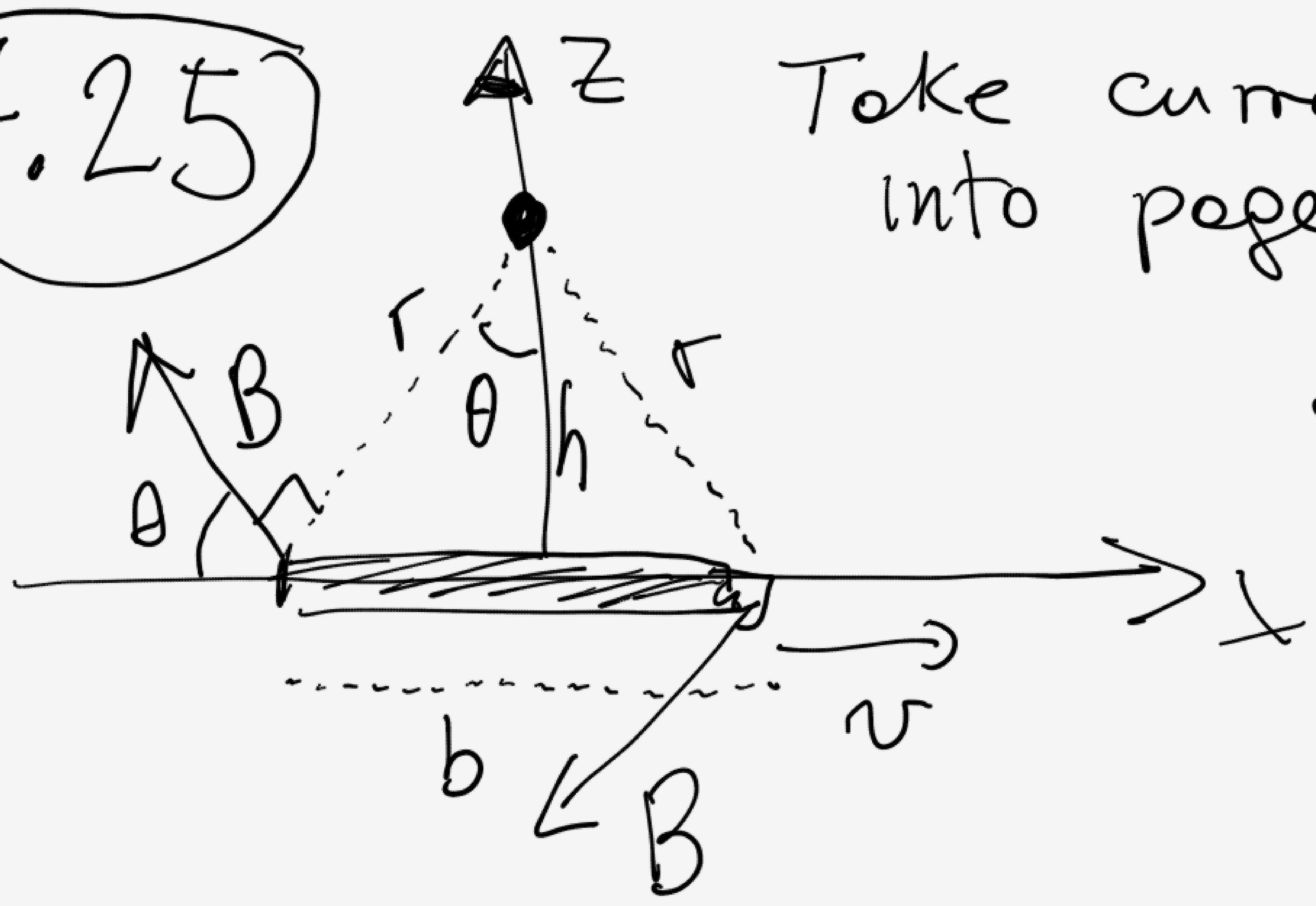
Amplitude of $E = E_0 = \dfrac{\mathcal{E}_0}{2\pi R} = \dfrac{\omega R B_0}{2}$

$$E_0 = \frac{2\pi}{2} \cdot 2.5 \cdot 10^6 \cdot 3\,10^{-2} \cdot 4\,10^{-4}\ V/m$$

$$\boxed{E_0 = 94\,\frac{V}{m}}$$

GO TO PAGE 8 FOR A JUSTIFICATION THAT $\vec{E}$ IS PURELY TANGENTIAL

7.25



The field at the edge of the loop is $B = \frac{\mu_0 I}{2\pi r}$. Only the z-component of this field contributes to ϕ $|B_z| = B\sin\theta$

But $\frac{b}{2} = r\sin\theta \Rightarrow |B_z| = \frac{\mu_0 I b}{4\pi r^2}$

But $r^2 = h^2 + b^2/4 \Rightarrow B_z = \frac{\mu_0 I b}{4\pi (h^2 + b^2/4)}$

We want $\frac{d\phi}{dt}$. In dt the loop will have moved an amount $v\,dt$ to the right. Since B_z has opposite sign on the two sides of the loop, in magnitude

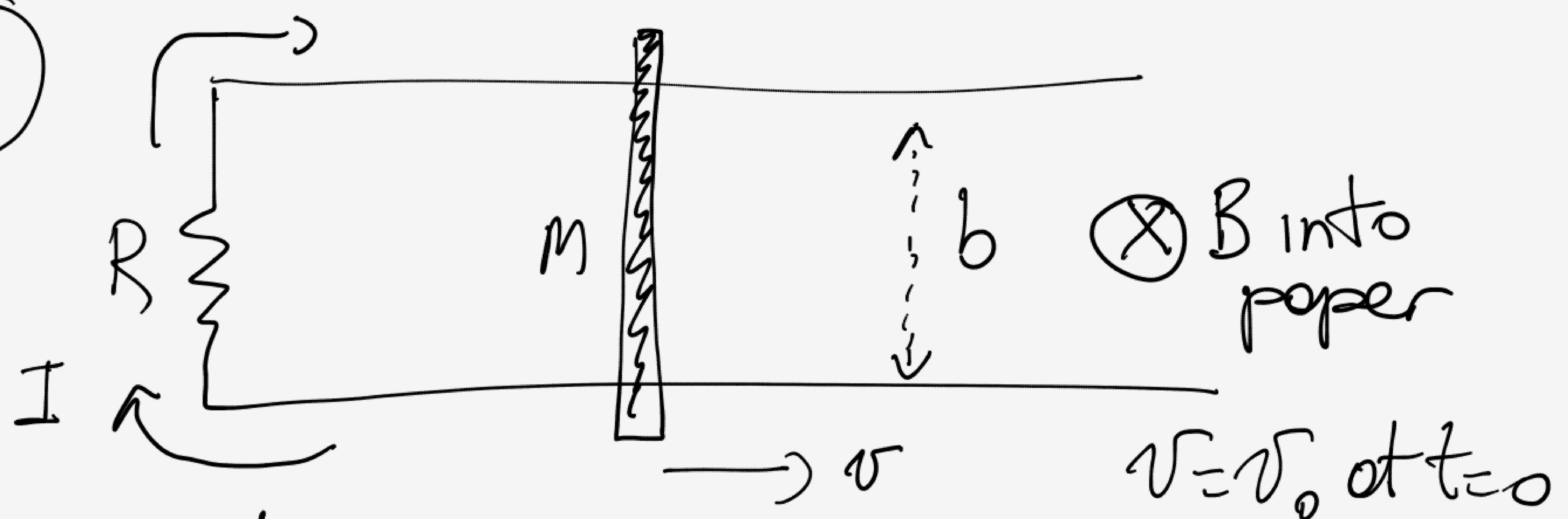
$$|d\phi| = 2\,|B_z|\,\underbrace{v\,dt\,b}$$

Area swept out by moving loop

$$|d\Phi| = 2\,\frac{\mu_0 I b}{4\pi\left(h^2+\frac{1}{4}b^2\right)}\, b\, v\, dt$$ (3)

$$\Rightarrow |\mathcal{E}| = \left|\frac{d\Phi}{dt}\right| = \frac{\mu_0 I b^2 v}{2\pi\left(h^2+\frac{1}{4}b^2\right)}$$

1.26



$$\frac{d\Phi}{dt} = v b B = \mathcal{E} \qquad I = \frac{\mathcal{E}}{R} = \frac{Bbv}{R}$$

Because the bar carries current I, there will be a force on the bar $F = IBb$

$F = \frac{B^2 b^2 v}{R}$ – Now we have to decide whether the force is going to be against the motion or in the direction of motion – Clearly physics tells us that it has to be against the motion, ie, in the direction of slowing down the rod – Otherwise we have just invented a perpetual motion machine (!) –

You can check the direction using right hand rules, etc.

So we have $m\frac{dv}{dt} = -\frac{B^2 b^2 v}{R}$

The solution is $v(t) = v_0 e^{-t/\tau}$

with $\boxed{\tau = \frac{mR}{B^2 b^2}}$

(a) The rod slows down but never stops moving ($v \to 0$ as $t \to \infty$)

(b) $v = \frac{dx}{dt} \Rightarrow x(t) = C - v_0 \tau e^{-t/\tau}$

where C is a constant. Now choose C such that $x(t=0) = x_0$

This gives $x_0 = C - v_0 \tau$ or

$C = x_0 + v_0 t$

$\Rightarrow \boxed{x(t) = x_0 + v_0 \tau (1 - e^{-t/\tau})}$

At $t = \infty$, $x(t) = x_0 + v_0 \tau$, therefore the rod will travel $\Delta x = v_0 \tau$

$$\boxed{\Delta x = \frac{v_0 m R}{B^2 b^2}}$$

(c) The kinetic energy lost by the rod, $\frac{1}{2}mV_0^2$ must have been dissipated in the resistor. Let's check that. Power in resistor $P = \frac{dE}{dt} = I^2R$

$$\frac{dE}{dt} = \left(\frac{Bbv}{R}\right)^2 R = \frac{B^2b^2v^2}{R} = \frac{B^2b^2}{R}V_0^2 e^{-2t/\tau}$$

Total energy dissipated is

$$E = \int_0^\infty \frac{B^2b^2}{R} V_0^2 e^{-2t/\tau}\,dt$$

$$E = \frac{\tau}{2}\frac{B^2b^2}{R}V_0^2 = \frac{mR}{2B^2b^2}\frac{B^2b^2}{R}V_0^2$$

$$E = \frac{1}{2}mV_0^2$$ ✓ as expected!!

7.27 (a) $B(t) = \mu_0 n I(t) = \mu_0 n I_0 \cos\omega t$

$$\Phi(t) = B(t)\pi r^2 = \mu_0 n I_0 \pi r^2 \cos\omega t$$

$$\boxed{\mathcal{E} = -\frac{d\Phi}{dt} = \mu_0 n I_0 \omega \pi r^2 \sin\omega t}$$

$$\boxed{I = \mathcal{E}/R = \frac{\mu_0 n I_0 \omega \pi r^2 \sin\omega t}{R}}$$

(b) $\vec{F} = I(\text{loop})\, d\vec{l} \times \vec{B}$ (6)

Where $d\vec{l}$ is along the ring -

The force is always radial -

Note that $d\vec{l}$ is always $\perp$ to B

So in magnitude

$$F = I(\text{loop})\, dl\, B =$$

$$F = I(\text{loop})\, dl\, \mu_0 n I_0 \cos\omega t$$

$$F = \frac{\pi r^2 \mu_0^2 n^2 I_0^2 \omega\, dl}{R} \sin\omega t \cos\omega t$$

But $\sin\omega t \cos\omega t = \frac{1}{2} \sin 2\omega t$

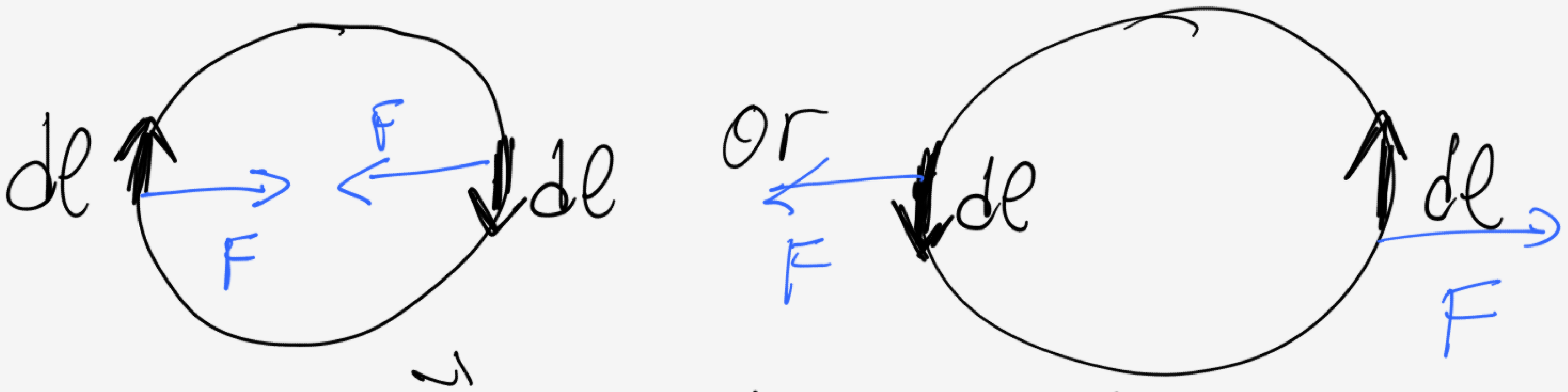
$$F = \frac{\pi r^2 \mu_0^2 n I_0^2 \omega\, dl}{2R} \sin 2\omega t$$

Force is maximum at

$$2\omega t = \pm \frac{\pi}{2} + m\pi$$

where m is an integer

Note. The force on opposite 7
sides of the ring is in opposite directions, eg either



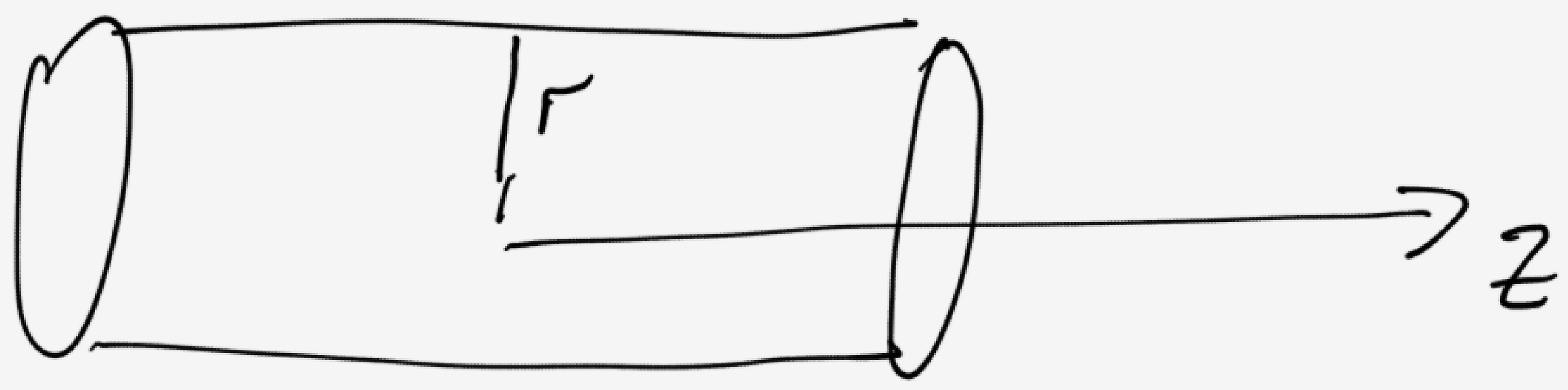
because $\vec{dl}$ points in different directions.

(C) The force tends to shrink or stretch the ring (it alternates between the two)

Addendum to 7.22 - (By CC and Dan P.) - We assumed that the field was totally tangential - Now we want to show it. We are going to work in cylindrical coordinates

$$\vec{E} = E_r \hat{r} + E_\theta \hat{\theta} + E_z \hat{z}$$

Consider gaussian surface which is a cylinder, on axis, length l



Gauss' law $\int \vec{E}\, d\vec{A} = 0 = E_r(r)\, 2\pi r \cdot l$

$$- \int E_z\left(z = -\tfrac{l}{2}\right) dA + \int E_z\left(z = \tfrac{l}{2}\right) dA = 0$$

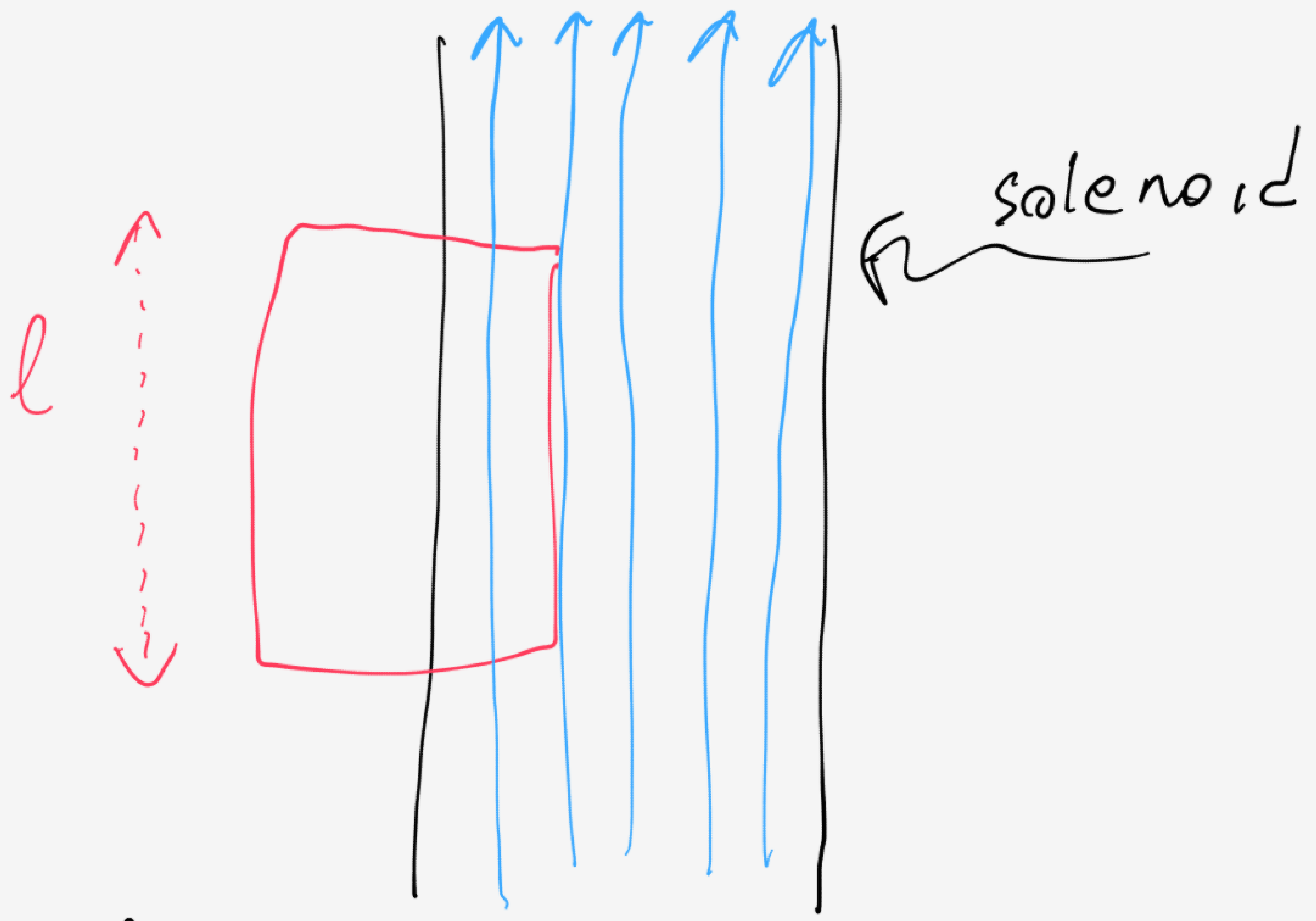
Since the solenoid is infinitely long E_z must be independent of z, therefore the last two integrals cancel - So we are left with $E_r(r)\, 2\pi r l = 0$

$$\Longrightarrow \boxed{E_r = 0}$$

NO RADIAL COMPONENT

Now let's look at E_z —

It cannot depend on θ by obvious symmetry. It cannot depend on z by the argument of the previous page (infinitely long solenoid). Now consider square path across solenoid



The flux is 0 through path

All fields must be zero outside solenoid

The radial $\vec{E}$ is 0 $\Rightarrow \oint \vec{E}\,d\vec{l}$ only has contribution from the segment inside solenoid parallel to z-axis. $\oint \vec{E}\,d\vec{l} = E_z l = 0 \Rightarrow \boxed{E_z = 0}$