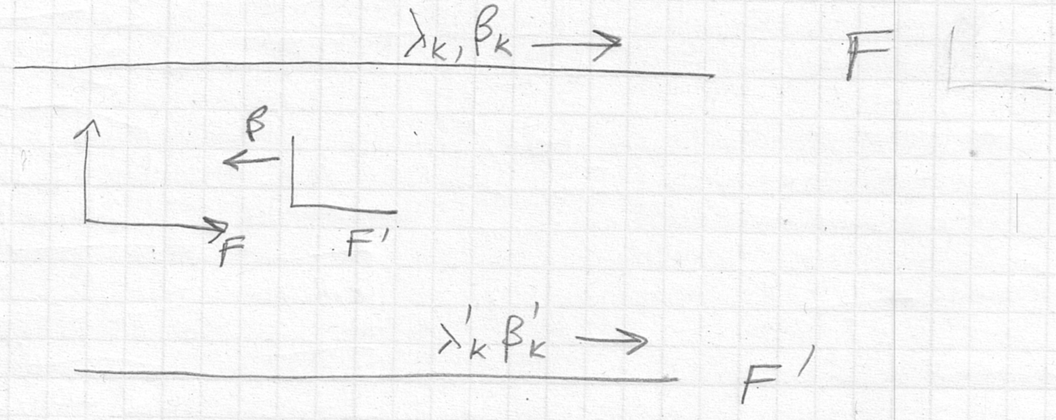
PHYSICS 24 – HOMEWORK 5

5.30



Following the discussion of section 5.9

$$\beta'_K = \frac{\beta_K + \beta}{1 + \beta_K \beta}$$

(same as equation for β'_0 in figure 5.22 but with a different sign because β is defined differently here wrt 5.22)

We will need $\gamma'_K = \dfrac{1}{\sqrt{1-\beta_K'^2}} = \left(1 - \dfrac{(\beta_K+\beta)^2}{(1+\beta_K\beta)^2}\right)^{-1/2}$

$$\gamma'_K = \left(\frac{1+\beta_K^2\beta^2 + 2\beta_K\beta - \beta_K^2 - \beta^2 - 2\beta_K\beta}{(1+\beta_K\beta)^2}\right)^{-1/2} = \left(\frac{(1-\beta_K^2)(1-\beta^2)}{(1+\beta_K\beta)^2}\right)^{-1/2}$$

$$\gamma'_K = (1+\beta_K\beta)\underbrace{\frac{1}{\sqrt{1-\beta_K^2}}}_{=\gamma_K}\ \underbrace{\frac{1}{\sqrt{1-\beta^2}}}_{=\gamma} \qquad \boxed{\gamma'_K = \gamma\gamma_K(1+\beta_K\beta)}$$

Let λ_K^0 be the linear charged density in rest frame of K-charges

$$\lambda_K = \gamma_K \lambda_K^0$$

Then the linear charged density in F' is $\lambda'_K = \gamma'_K \lambda_K^0$

$$\lambda'_K = \gamma'_K\left(\frac{\lambda_K}{\gamma_K}\right) = \frac{\gamma'_K}{\gamma_K}\lambda_K = \frac{\gamma\gamma_K(1+\beta_K\beta)}{\gamma_K}\lambda_K = \gamma(1+\beta_K\beta)\lambda_K$$

$\lambda'_K = \gamma(\lambda_K + \beta\beta_K\lambda_K)$ But $I_K = \lambda_K \beta_K c$

$\Rightarrow \boxed{\lambda'_K = \gamma\left(\lambda_K + \beta \frac{I_K}{c}\right)}$ ✓

6.29 The force is always $\perp$ to $\vec{v}$. $\vec{F} = q\vec{v}\times\vec{B}$

In magnitude, the force is $F = qvB$

A $\perp$ force does no work, therefore does not change the energy, therefore v = constant (in magnitude)

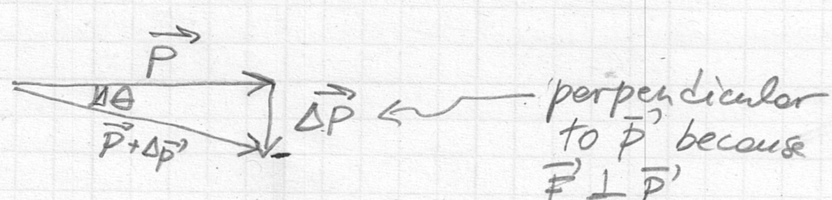
Going back to classical physics from last year, you would then have <u>circular motion</u> with centripetal force $\frac{mv^2}{R} = qvB$ This would give $R = \frac{mv}{qB}$

But $mv = p$ (in classical physics), so $R = \frac{mv}{qB} = \frac{p}{qB}$

But in relativity $p = \gamma m v$, so it is not clear what is correct anymore. Let's start from first principles

We are going to do this in two ways

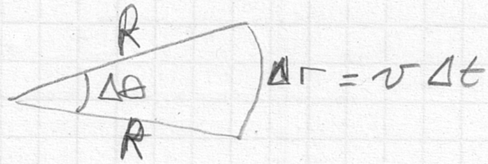
<u>First method</u> Follow the book's hint $\Delta p = qvB\Delta t$



In magnitude, $|\vec{p}+\Delta\vec{p}| = |\vec{p}| = p$

$\Delta\theta \sim \frac{\Delta p}{p} = \frac{qvB\Delta t}{p}$

Going now from "momentum space" to real space we get this picture



$\Delta r = R\,\Delta\theta$

$v\Delta t = R\,\dfrac{qvB\Delta t}{p}$

$$\boxed{R = \frac{p}{qB} = \frac{\gamma m v}{qB}}$$

Second method

$$\frac{d\vec{p}}{dt} = q\vec{v}\times\vec{B} \qquad \gamma m \frac{d\vec{v}}{dt} = q\vec{v}\times\vec{B}$$

(note $\gamma = \dfrac{1}{\sqrt{1-v^2/c^2}}$ but v^2 is constant)

Let $\vec{B} = B\hat{z}$ Then $\vec{v}\times\vec{B} = v_y\hat{x} - v_x\hat{y}$

$$\frac{dv_x}{dt} = \frac{qB}{\gamma m} v_y \quad \text{and} \quad \frac{dv_y}{dt} = -\frac{qB}{\gamma m} v_x$$

Take derivative of first equation

$$\frac{d^2v_x}{dt^2} = \frac{qB}{\gamma m}\frac{dv_y}{dt} \qquad \text{substitute } \frac{dv_y}{dt} = \frac{qB}{\gamma m} v_x$$

$$\frac{d^2v_x}{dt^2} = -\frac{q^2B^2}{\gamma^2 m^2} v_x$$

This is like simple harmonic motion

$$v_x = A\cos(\omega t + \phi) \qquad \text{with } \omega = \frac{qB}{\gamma m}$$

Also $\dfrac{dv_y}{dt} = -\dfrac{qB}{\gamma m} v_x = -\omega A(\cos\omega t + \phi)$

gives $v_y = -A\sin(\omega t + \phi)$

Now $V_x^2 + V_y^2 = V^2$ results in $A = v = \frac{p}{\gamma m}$

$$\begin{cases} V_x = V \cos(\omega t + \phi) = \frac{p}{\gamma m} \cos(\omega t + \phi) \\ V_y = -V \sin(\omega t + \phi) = -\frac{p}{\gamma m} \sin(\omega t + \phi) \end{cases}$$

But since $V_x = \frac{dx}{dt}$ and $V_y = \frac{dy}{dt}$ we get

$$\begin{cases} x = \frac{p}{\gamma m \omega} \sin(\omega t + \phi) \\ y = \frac{p}{\gamma m \omega} \cos(\omega t + \phi) \end{cases}$$

This is the equation of a circle of radius $R = \frac{p}{\gamma m \omega} = \frac{p}{\gamma m} \frac{\gamma m}{qB}$

$$\boxed{R = \frac{p}{qB} = \frac{\gamma m v}{qB}}$$

We can get the time for one revolution as

$$\Delta T = \frac{2\pi R}{v} = \frac{2\pi}{v} \frac{\gamma m v}{qB} = 2\pi \frac{\gamma m}{qB}.$$

or, starting from $\omega = \frac{qB}{\gamma m}$ and $\omega = \frac{2\pi}{T} \Rightarrow T = \frac{2\pi \gamma m}{qB}$

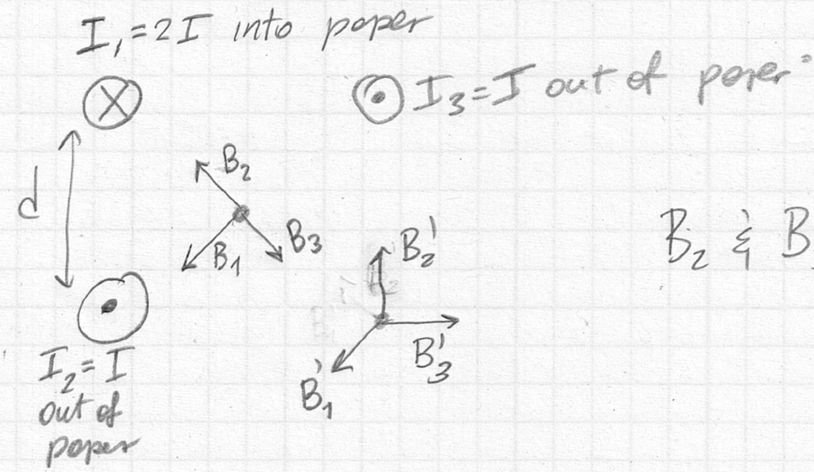
6.30 The speed is $\sim c$ (good enough approximation)

$$R \cong \frac{p}{qB} \cong \frac{\gamma m c}{qB} = \frac{10^7 \; 1.67 \; 10^{-27} \, kg \; 3 \; 10^8 \, m/sec}{1.6 \; 10^{-19} \, C \; 3 \cdot 10^{-10} \, T}$$

$$\boxed{R = 1.0 \; 10^{17} \, m}$$

$$T = \frac{2\pi R}{v} \simeq \frac{2\pi \; 1 \; 10^{17} \, m}{3 \; 10^8 \, m/sec} = \underline{2.1 \; 10^9 \, sec}$$

6.31



B_2 & B_3 cancel

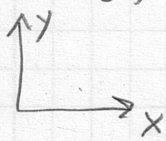
$$B_1 = \frac{\mu_0}{2\pi}\frac{I_1}{d/\sqrt{2}} = \frac{\mu_0}{2\pi}\sqrt{2}\,\frac{2I}{d} = \frac{\sqrt{2}\mu_0 I}{\pi d}$$ field at center

At corner, the situation is a bit more complicated

In magnitude
$$B_1' = \frac{\mu_0}{2\pi}\frac{2I}{\sqrt{2}d} = \frac{\mu_0}{2\pi}\frac{\sqrt{2}I}{d}$$

$$B_2' = B_3' = \frac{\mu_0 I}{2\pi d}$$

We need to add the three fields vectorially, therefore we need to look at components



$$B_1' = \frac{\mu_0 I}{2\pi}(-1, -1, 0)$$

$$B_2' = \frac{\mu_0 I}{2\pi}(0, 1, 0)$$

$$B_3' = \frac{\mu_0 I}{2\pi}(1, 0, 0)$$

If we add these fields vectorially, we get $\underline{B=0}$ at corner