PHYSICS 24
HOMEWORK SET 3

(1) $E = \gamma m_0 c^2 = \frac{1}{\sqrt{1-v^2/c^2}} m_0 c^2$

$\sqrt{1-\frac{v^2}{c^2}} = \frac{m_0 c^2}{E}$ $\qquad 1-\frac{v^2}{c^2} = \frac{m_0^2 c^4}{E^2}$

$\frac{v}{c} = \sqrt{1-\frac{m_0^2 c^4}{E^2}} = \sqrt{1-\frac{0.938^2\ 10^{18}\ eV^2}{4^2\ 10^{24}\ eV^2}} = \underline{\underline{0.99999973}}$

(2) K&K 13.2

(a) $\frac{K_{REL}}{K_{CL}} = \frac{(\gamma-1) m_0 c^2}{\frac{1}{2} m_0 v^2} = 2(\gamma-1)\frac{c^2}{v^2}$

$\gamma = \frac{1}{\sqrt{1-v^2/c^2}} = \left(1-\frac{v^2}{c^2}\right)^{-1/2}$ $\qquad$ Use $(1+x)^{-1/2} \cong 1-\frac{1}{2}x+\frac{3}{8}x^2$

$\gamma \approx 1+\frac{1}{2}\frac{v^2}{c^2}+\frac{3}{8}\frac{v^4}{c^4}$

$\Rightarrow \frac{K_{REL}}{K_{CL}} \approx 2\left(1+\frac{1}{2}\frac{v^2}{c^2}+\frac{3}{8}\frac{v^4}{c^4}-1\right)\frac{c^2}{v^2} = \left(\frac{v^2}{c^2}+\frac{3}{4}\frac{v^4}{c^4}\right)\frac{c^2}{v^2}$

$\frac{K_{REL}}{K_{CL}} \approx 1+\frac{3}{4}\frac{v^2}{c^2}$ $\qquad$ 10% difference when $\frac{3}{4}\frac{v^2}{c^2} = \frac{1}{10}$

$\boxed{\frac{v^2}{c^2} = \frac{2}{15}}$

(b) $\gamma = \frac{1}{\sqrt{1-2/15}} = \sqrt{\frac{15}{13}} = \underline{\underline{1.07}}$ $\qquad K = (\gamma-1) m_0 c^2 \approx 0.07 m_0 c^2$

For electrons $\quad K = 0.07 \times 0.51\ MeV = 0.036\ MeV$

For protons $\quad K = 0.07 \times 930\ MeV = \underline{65\ MeV}$

3) K&K 13.4

For each particle $E = \gamma m_0 c^2$

For particle 1 $\vec{P}_1 = \gamma m_0 v \hat{x}$

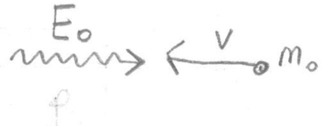
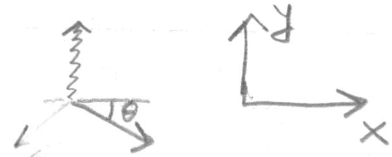
Do a Lorentz transformation in rest system of 2

$$E_1' = \gamma (E_1 + v P_{1x}) = \gamma(\gamma m_0 c^2 + \gamma m_0 v^2)$$

$$E_1' = \gamma^2 m_0 c^2 \left(1 + \frac{v^2}{c^2}\right) = \frac{1+v^2/c^2}{1-v^2/c^2} m_0 c^2$$

(Check book clue: if $v^2/c^2 = 1/2$, $E_1' = \frac{3/2}{1/2} m_0 c^2 = 3 m_0 c^2$)

4) K&K 13.8 (a)

Before  After 

Look at the 4-vectors before and after

I will write the 4-vectors as $(P_x\ P_y\ P_z\ E)$.

I will set $c=1$ in the algebra below

Before

$$\underset{\sim}{P_\gamma} = (E_0, 0, 0, E_0) \qquad \underset{\sim}{P_m} = (-\gamma m_0 v, 0, 0, \gamma m_0)$$

$$\Rightarrow \underset{\sim}{P_{before}} = \underset{\sim}{P_\gamma} + \underset{\sim}{P_m} = (E_0 - \gamma m_0 v, 0, 0, E_0 + \gamma m_0)$$

After

$$\underset{\sim}{P_\gamma} = (0, E, 0, E) \qquad \underset{\sim}{P_m} = (\gamma' m_0 v' \cos\theta, -\gamma' m_0 v' \sin\theta, 0, \gamma' m_0)$$

velocity of the electron after the collision

Here $\gamma' = \dfrac{1}{\sqrt{1-\frac{v'^2}{c^2}}}$

$\underset{\sim}{P}_{after} = (\gamma m_0 v' \cos\theta,\ E - \gamma' m_0 v' \sin\theta,\ 0,\ \gamma' m_0 + E)$

Conservation of 4-momentum. Use book notation

$E_i = \gamma m_0 c^2$.

$$\begin{cases} E_0 - v E_i = \gamma' m_0 v' \cos\theta & (1) \\ 0 = E - \gamma' m_0 v' \sin\theta & (2) \\ E_0 + E_i = \gamma' m_0 + E & (3) \end{cases}$$

Three equations. Three unknowns. <u>From now on, just algebra</u>

(v', θ, E)

Rewrite (1) and (2) as

$$\begin{cases} E_0 - v E_i = \gamma' m_0 v' \cos\theta \\ E = \gamma' m_0 v' \sin\theta \end{cases}$$

Square both equations and sum. Use $\sin^2\theta + \cos^2\theta = 1$

$$(E_0 - v E_i)^2 + E^2 = \gamma'^2 m_0^2 v'^2 = |\vec{P}|^2 = \gamma'^2 m_0^2 - m_0^2$$

momentum of electron after collision

$$\Rightarrow \gamma'^2 m_0^2 = (E_0 - v E_i)^2 + E^2 + m_0^2 \quad (4)$$

But equation (3) is $\gamma' m_0 = E_0 + E_i - E$

Taking the square of this, and plugging into (4)

$$(E_0 + E_i - E)^2 = (E_0 - v E_i)^2 + E^2 + m_0^2$$

$$\cancel{E_0^2} + E_i^2 + \cancel{E^2} + 2E_0E_i - 2E(E_0 + E_i) = \cancel{E_0^2} + v^2 E_i^2 - 2E_0 E_i v + \cancel{E^2} + m_0^2$$

$$(1 - v^2) E_i^2 + 2E_0E_i(1 + v) = 2E(E_0 + E_i) + m_0^2$$

But $(1-v^2)E_i^2 = (1-v^2)\gamma^2 m_0^2 = \frac{1-v^2}{1-v^2} m_0^2 = m_0^2$

(Remember, I had set $c=1$)

Therefore the eqtn at the bottom of the last page becomes

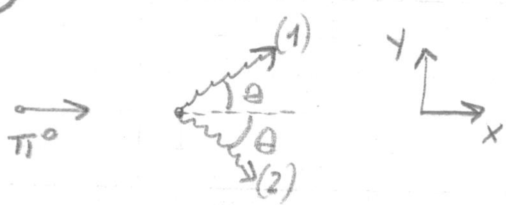
$$\cancel{m_0^2} + 2E_0E_i(1+v) = 2E(E_0+E_i) + \cancel{m_0^2}$$

$$\boxed{E = \frac{E_0E_i(1+v)}{E_0+E_i} = \frac{E_0(1+v/c)}{(1+E_0/E_i)}}$$

(I have restored the appropriate factor of c at the very end)

K&K 14.1

5 I will set $c=1$ in the algebra



$E = 100$ MeV $|\vec{P}_1| = |\vec{P}_2| = E$

$\underset{\sim}{P_1} = (E\cos\theta, E\sin\theta, 0, E)$

$\underset{\sim}{P_2} = (E\cos\theta, -E\sin\theta, 0, E)$

$(\underset{\sim}{P_1} + \underset{\sim}{P_2}) = 2E(\cos\theta, 0, 0, 1)$

$(\underset{\sim}{P_1} + \underset{\sim}{P_2})^2 = M_\pi^2 \Rightarrow 4E^2(1-\cos^2\theta) = M_\pi^2 \Rightarrow 2E\sin\theta = M_\pi$

$\sin\theta = \frac{M_\pi}{2E} = \frac{135}{200} = 0.675$ $\boxed{\theta \approx 42^\circ}$

The answer to part (a) is easy — Conservation of energy

$\gamma m_\pi = 2E$ $\quad \frac{1}{\gamma^2} = \frac{m_\pi^2}{4E^2}$ $\quad 1 - \frac{v^2}{c^2} = \frac{m_\pi^2}{4E}$ $\quad \frac{v}{c} = \sqrt{1 - \frac{M_\pi^2}{4E^2}}$

$\boxed{v/c \cong 0.74}$

6.6 K&K 14.2

Set $c=1$

At threshold in the CM frame, the p and π^0 are at rest

$$\underset{\sim}{P}_{after} = (\vec{0}, m_p + m_\pi) \Rightarrow \underset{\sim}{P}_{after}^2 = (m_\pi + m_p)^2$$

They must be the same

In lab frame, before collision

$$\underset{\sim}{P}_\gamma = (E, 0, 0, E) \qquad \underset{\sim}{P}_p = (0, 0, 0, m_p)$$

$$\underset{\sim}{P}_{before} = (E, 0, 0, E + m_p) \Rightarrow \underset{\sim}{P}_{before}^2 = (E+m_p)^2 - E^2$$

$$(E+m_p)^2 - E^2 = (m_\pi + m_p)^2$$

$$\cancel{E^2} + 2Em_p + m_p^2 - \cancel{E^2} = (m_\pi + m_p)^2$$

$$\boxed{E = \frac{(m_\pi + m_p)^2 - m_p^2}{2m_p}}$$

$$E = \frac{(938+135)^2 - 938^2}{2 \cdot 938} \text{ MeV} = \underline{145 \text{ MeV}}$$