PHYSICS 24 FINAL

1

RL in series $\tilde{Z} = R + i\omega L = \sqrt{R^2+\omega^2L^2}\, e^{i\phi}$ $\boxed{\phi = \tan^{-1}\frac{\omega L}{R}}$

$$\tilde{I}_1 = \frac{\tilde{V}}{\tilde{Z}} = \frac{\mathcal{E}_0 e^{i\omega t}}{\sqrt{R^2+\omega^2L^2}\, e^{i\phi}} = \frac{\mathcal{E}_0}{\sqrt{R^2+\omega^2 L}}\, e^{i(\omega t-\phi)}$$

Taking the real part $\boxed{I_1 = \frac{\mathcal{E}_0}{\sqrt{R^2+\omega^2L^2}} \cos(\omega t - \phi)}$

$$C : \tilde{Z}_C = \frac{1}{i\omega C} \qquad \tilde{I}_2 = \frac{\tilde{V}}{\tilde{Z}_C} = \frac{\mathcal{E}_0 e^{i\omega t}}{1/\omega C}\, i$$

But $i = e^{i\frac{\pi}{2}}$ $\qquad \tilde{I}_2 = \omega C \mathcal{E}_0\, e^{i(\omega t + \frac{\pi}{2})}$

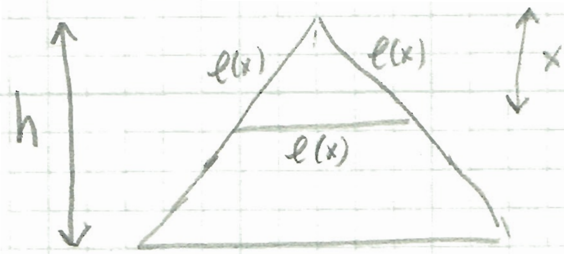
Taking the real part

$$\boxed{I_2 = \omega C \mathcal{E}_0 \cos(\omega t + \tfrac{\pi}{2}) = -\omega C \sin \omega t}$$

(2) All angles in the triangle are 60°.

$\Rightarrow$ Height of triangle $h = a \sin 60^\circ = \frac{\sqrt{3}}{2} a$

Picture of triangle



$x = \ell \sin 60^\circ$

$\Rightarrow \ell = \frac{2}{\sqrt{3}} x$

$B(x) = \frac{\mu_0 I}{2\pi (x+b)}$ ($x+b$ is the distance from the wire)

Flux through a small horizontal strip at x : $d\Phi = B(x)\,\ell\,dx$

But since $\ell = \frac{2}{\sqrt{3}} x$, $d\Phi = \frac{2B(x)}{\sqrt{3}} x\,dx = \frac{\mu_0 I}{\sqrt{3}\pi} \frac{x}{x+b} dx$

Total flux $\Phi = \frac{\mu_0 I}{\sqrt{3}\pi} \int_0^h \frac{x}{x+b} dx = \frac{\mu_0 I}{\sqrt{3}\pi} \int_0^h \frac{x+b-b}{x+b} dx$

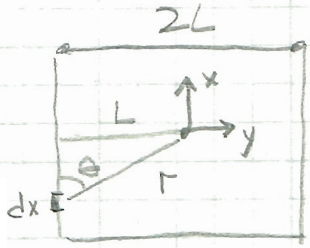
$$\Phi = \frac{\mu_0 I}{\sqrt{3}\pi} \int_0^h dx - \frac{\mu_0 I b}{\sqrt{3}\pi} \int_0^h \frac{dx}{x+b}$$

$$\Phi = \frac{\mu_0 I}{\sqrt{3}\pi} h - \frac{\mu_0 I b}{\sqrt{3}\pi} \log \frac{h+b}{b}$$

And since $h = \frac{\sqrt{3}}{2} a$ $\Phi = \frac{\mu_0 I}{\pi} \left[\frac{a}{2} - \frac{b}{\sqrt{3}} \log \frac{\frac{\sqrt{3}}{2} a + b}{b} \right]$

And finally $\boxed{M = \Phi / I = \frac{\mu_0}{\pi} \left[\frac{a}{2} - \frac{b}{\sqrt{3}} \log \frac{\frac{\sqrt{3}}{2} a + b}{b} \right]}$

3



Find B from one side, then multiply by 4

$$dB = \frac{\mu_0 I}{4\pi} \frac{\vec{dx} \times \hat{r}}{r^2}$$

$r^2 = x^2 + L^2 \qquad L = r\sin\theta$

$|\vec{dx} \times \hat{r}| = \sin\theta\, dx = \frac{L\, dx}{r}$

$$dB = \frac{\mu_0 I}{4\pi} \frac{L\, dx}{r^3}$$

$$B_{TOTAL} = \underbrace{4}_{\text{4 sides}} \cdot \frac{\mu_0 I L}{4\pi} \int_{-L}^{L} \frac{dx}{(x^2+L^2)^{3/2}}$$

Use eqtn (41) in the pages of integral provided

$$B_{TOTAL} = \frac{\mu_0 I L}{\pi} \left[\frac{x}{L^2\sqrt{x^2+L^2}} \right] = \frac{\mu_0 I L}{\pi} \left[\frac{2L}{L^2 \sqrt{2L^2}} \right]$$

$$\boxed{B_{TOTAL} = \frac{\sqrt{2}\, \mu_0 I}{\pi L}}$$

4

(a) By conservation of momentum the electron and photon will have equal and opposite momenta. $\boxed{\text{Use } c=1}$. P = momentum

Conservation of energy

$$m_\mu = \sqrt{m_e^2 + P^2} + P$$

$$m_\mu - P = \sqrt{m_e^2 + p^2}$$

$$m_\mu^2 - 2m_\mu P + \cancel{p^2} = m_e^2 + \cancel{p^2}$$

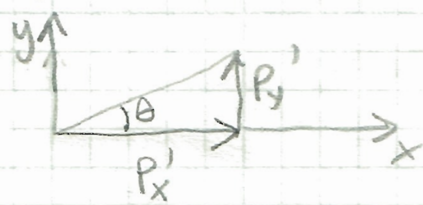
$$\boxed{p = \frac{m_\mu^2 - m_e^2}{2m_\mu}}$$

(b) $q = \gamma m_\mu \beta \Rightarrow \gamma\beta = 10$. This is the $\gamma\beta$ of the boost.

In the new frame

$$P_y' = P_y = P$$

$$P_x' = \gamma(P_x + \beta E) = \gamma(0 + \beta P) = \gamma\beta P = 10P$$



$$\theta = \tan^{-1}\frac{P_y'}{P_x'} \qquad \boxed{\theta = \tan^{-1}\frac{1}{10}}$$

5 (a) $d_{BOB} = 0.8 \times 30$ years $=$ 24 light-years

(b) $\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-(4/5)^2}} = \frac{5}{3}$

Time dilation: $t_{BOB} = \gamma t_{MARTHA}$

$t_{MARTHA} = \frac{1}{\gamma} t_{BOB} \Rightarrow t_{MARTHA} = \frac{3}{5} 30 = $ 18 years

(c) M. sees Earth and Zorg passing by her traveling at 0.8c, separated by 18 years

$\Rightarrow d_{MARTHA} = v\, t_{MARTHA} = 0.8c \cdot 18$ years $=$ 14.4 light years

(d) Invariance of event intervals.

$c^2 t_{BOB}^2 - d_{BOB}^2 = c^2 t_{RALPH}^2 - d_{RALPH}^2$

$d_{RALPH}^2 = 25^2 + 24^2 - 30^2$ (light-years)^2

$d_{RALPH} = 17.3$ light-years

Or, from the point of view of MARTHA both events, (her passing by earth, and her reaching Zorg) happen at the same place and are separated by 18 light years

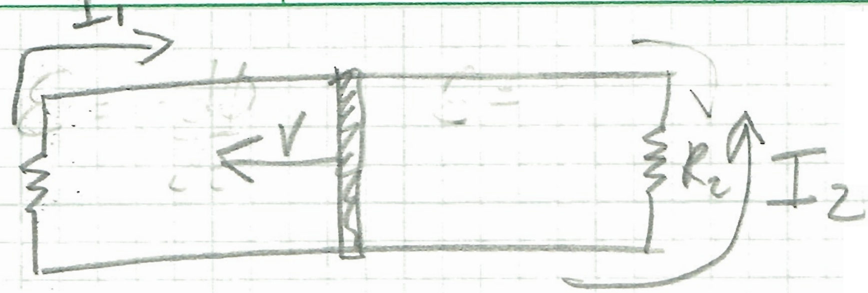
$ct_{MARTHA}^2 - 0 = c^2 t_{RALPH}^2 - d_{RALPH}^2$

$d_{RALPH}^2 = 25^2 - 18^2$ (light years)^2

$d_{RALPH} = 17.3$ light-years

6

(a)



(b) $\mathcal{E} = -\frac{d\phi}{dt}$ $\qquad |\mathcal{E}| = BLv$

$$I_1 = \frac{\mathcal{E}}{R_1} = \frac{BLv}{R_1} \qquad I_2 = \frac{\mathcal{E}}{R_2} = \frac{BLv}{R_2}$$

(c) $\text{Power} = I_1^2 R_1 + I_2^2 R_2 = \underline{B^2 L^2 v^2 \left[\frac{1}{R_1} + \frac{1}{R_2}\right]}$

(d) By conservation of energy

$\text{Power} = Fv \Rightarrow \boxed{F = B^2 L^2 v \left[\frac{1}{R_1} + \frac{1}{R_2}\right]}$

Alternatively, total current through rod is

$I = I_1 + I_2$

Magnetic force $F = ILB = \left(\frac{BLv}{R_1} + \frac{BLv}{R_2}\right) LB$

$F = B^2 L^2 v \left[\frac{1}{R_1} + \frac{1}{R_2}\right]$ Same in magnitude

And this force must be be balanced by the external force.