S' = rest frame of particle _ Force $\vec{F}'$

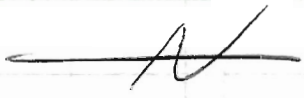
In a frame S where the particle has $\vec{v}$, the force $\vec{F}$ is such that

$$\vec{F}_{\parallel} = \vec{F}'_{\parallel} \quad \text{and} \quad \vec{F}_{\perp} = \frac{1}{\gamma}\vec{F}'_{\perp} \qquad \gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

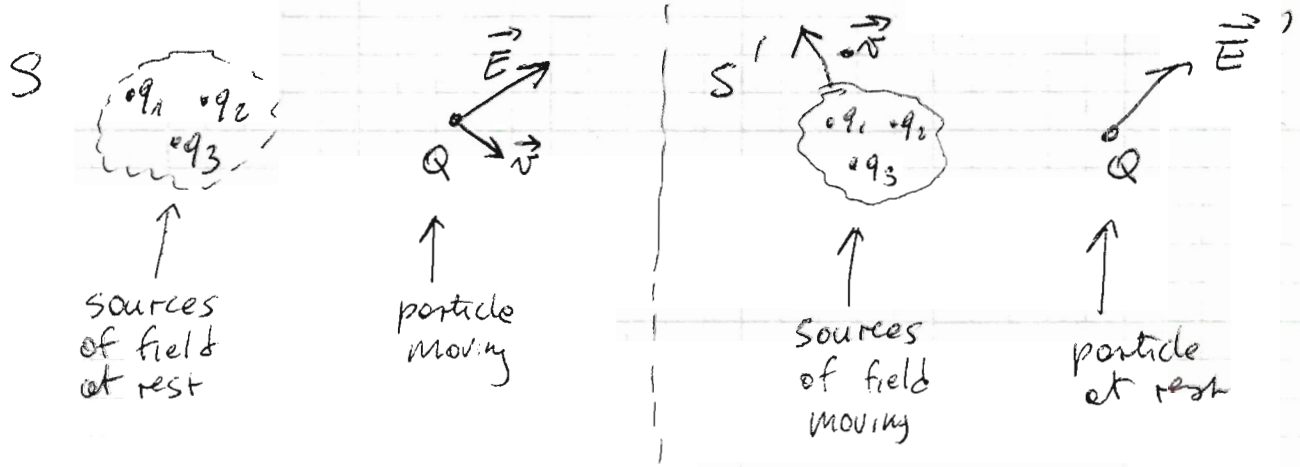
and $\parallel$, $\perp$ means parallel or perpendicular to $\vec{v}$

This is equations 6.16 and 6.17 in Purcell

(But careful because in Purcell the unprimed frame is the frame where the particle is at rest_



Now consider these two situations



From Purcell $\vec{E}'_{\parallel} = \vec{E}_{\parallel}$ $\quad \vec{E}'_{\perp} = \gamma \vec{E}_{\perp}$ $\quad$ Eqtn 5.7

In S' $\vec{F}' = Q\vec{E}'$ _ This defines $\vec{E}$. It is a vector such that a charge Q at rest will feel a force Q times the electric field_

Then $\vec{F}' = Q\vec{E}'$

$\vec{F}'_{\parallel} = Q\vec{E}'_{\parallel}$ and $\vec{F}'_{\perp} = Q\vec{E}'_{\perp}$

$\vec{F}_{\parallel} = Q\vec{E}'_{\parallel}$ and $\gamma\vec{F}_{\perp} = Q\vec{E}'_{\perp}$ (from eqtn G.16, G.17)

Then use Eqtn 5.7

$\vec{F}_{\parallel} = Q\vec{E}'_{\parallel}$ and $\gamma\vec{F}_{\perp} = \gamma Q\vec{E}_{\perp}$

$\Rightarrow \vec{F} = Q\vec{E}$ also for moving charges if sources are at rest