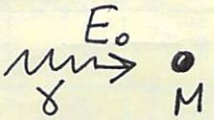
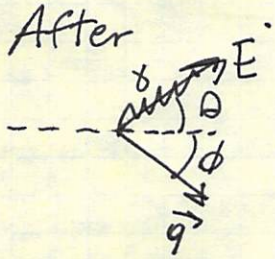
Compton Scattering

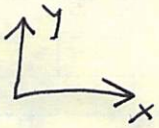
Scattering of photons off electrons

Before



After





Photon energy E_0
Electron at rest

Photon energy E, angle θ
Electron momentum $\vec{q}$, angle ϕ

What is the energy (E) of photons scattered at some given angle θ? | Set $c=1$ |

4-momentum Before

$$\underset{\sim}{P}_\gamma = (E_0, 0, 0, E_0) \quad \leftarrow \text{note: } m_\gamma = 0 \Rightarrow P_\gamma = E_\gamma$$

$$\underset{\sim}{P}_e = (0, 0, 0, M)$$

$$\boxed{\underset{\sim}{P}_{BEFORE} = (E_0, 0, 0, E_0 + M)}$$

4 momentum After

$$\underset{\sim}{P}_\gamma = (E\cos\theta, E\sin\theta, 0, E)$$

$$\underset{\sim}{P}_e = (q\cos\phi, -q\sin\phi, 0, \sqrt{q^2+M^2})$$

$$\underline{\underset{\sim}{P}_{AFTER} = (E\cos\theta + q\cos\phi, E\sin\theta - q\sin\phi, 0, E + \sqrt{q^2+M^2})}$$

Conservation of 4-momentum $\underset{\sim}{P}_{BEFORE} = \underset{\sim}{P}_{AFTER}$

$$\begin{cases} E_0 = E\cos\theta + q\cos\phi \quad (1) \\ 0 = E\sin\theta - q\sin\phi \quad (2) \\ E_0 + M = E + \sqrt{q^2 + M^2} \quad (3) \end{cases}$$

3 equations, 3 unknowns! E, q, ϕ

First: eliminate ϕ

(1): $q\cos\phi = E_0 - E\cos\theta$

(2): $q\sin\phi = E\sin\theta$

Take $(1)^2 + (2)^2$: $q^2\cos^2\phi + q^2\sin^2\phi = (E_0 - E\cos\theta)^2 + E^2\sin^2\theta$

$q^2 = E_0^2 + E^2\cos^2\theta - 2EE_0\cos\theta + E^2\sin\theta = E_0^2 + E^2 - 2EE_0\cos\theta$

Substitute q^2 into (3)

$$E_0 + M = E + \sqrt{E_0^2 + E^2 - 2EE_0\cos\theta + M^2}$$

$$(E_0 + M - E)^2 = E_0^2 + E^2 - 2EE_0\cos\theta + M^2$$

$$E_0^2 + M^2 + E^2 - 2E(E_0 + M) + 2E_0M = E_0^2 + E^2 - 2EE_0\cos\theta + M^2$$

$$E(E_0 + M - E_0\cos\theta) = E_0M$$

$$E = \frac{E_0M}{M + E_0(1 - \cos\theta)}$$

$$\boxed{E = \frac{E_0}{1 + \frac{E_0}{M}(1 - \cos\theta)}}$$

Restore factors of c

$$\boxed{E = \frac{E_0}{1 + \frac{E_0}{Mc^2}(1 - \cos\theta)}}$$