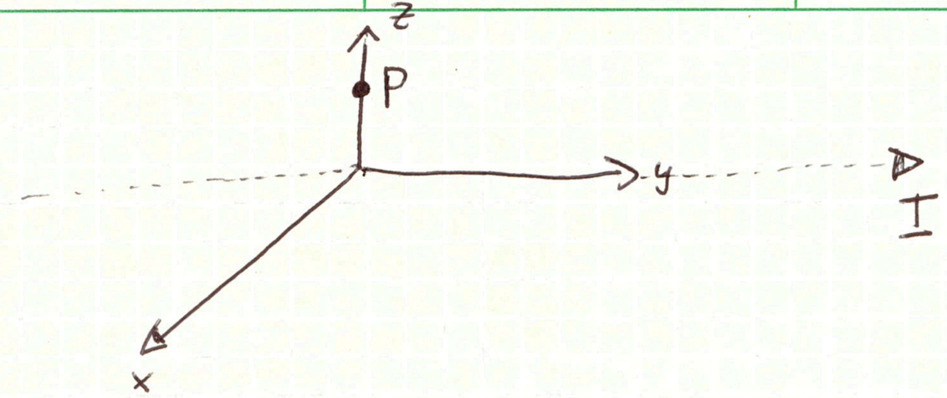
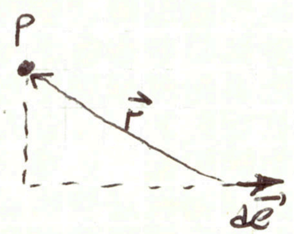


$P(x=0\ \ y=0\ \ z) \leftarrow$ Want $\vec{B}$ at this point

By Ampere's law $\quad \vec{B} = +\dfrac{\mu_0 I}{2\pi z}\,\hat{x}$

Can also be obtained by integrating along the core



$$d\vec{B} = \frac{\mu_0 I}{4\pi}\,\frac{d\vec{l} \times \hat{r}}{r^2} \qquad\qquad d\vec{l} = dy\ \hat{y}$$

$$\vec{r} = -y\,\hat{y} + z\,\hat{z} \qquad\qquad r = \sqrt{y^2+z^2}$$

$$\hat{r} = -\frac{y}{r}\,\hat{y} + \frac{z}{r}\,\hat{z}$$

$$d\vec{l} \times \hat{r} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & dy & 0 \\ 0 & -\frac{y}{r} & \frac{z}{r} \end{vmatrix} = \frac{z\,dy}{r}\,\hat{x}$$

$$d\vec{B} = \frac{\mu_0 I}{4\pi}\,\hat{x}\,\frac{z}{r^3}\,dy \qquad \cancel{\vec{B} = \frac{\mu_0 I z}{4\pi}\,\hat{x}\int_{-\infty}^{\infty}\frac{dy}{\sqrt{y^2+z}}}$$

$$\vec{B} = \frac{\mu_0 I z}{4\pi}\,\hat{x}\int_{-\infty}^{\infty}\frac{dy}{(y^2+z^2)^{3/2}} = \frac{\mu_0 I z}{4\pi}\left[\frac{y}{z^2\sqrt{z^2+y^2}}\right]_{-\infty}^{\infty}\hat{x}$$

$$\vec{B} = \frac{\mu_0 I z}{4\pi}\left[\frac{2}{z^2}\right]\hat{x} \qquad \boxed{\vec{B} = \frac{\mu_0 I}{2\pi z}\hat{x}}$$

Can also do via $\vec{A}$ instead of $\vec{B}$

$$d\vec{A} = \frac{\mu_0 I}{4\pi}\frac{d\vec{l}}{r} \qquad \vec{A} = \frac{\mu_0 I}{4\pi}\hat{y}\int_{-\infty}^{\infty}\frac{dy}{\sqrt{y^2+z^2}} = \frac{\mu_0 I}{2\pi}\hat{y}\int_0^{\infty}\frac{dy}{\sqrt{y^2+z^2}}$$

Since $\int\frac{dx}{\sqrt{x^2+a^2}} = \log|x+\sqrt{x^2+a^2}|$ this looks like a disaster because the integral diverges

But do it with an upper limit of integration instead

$$\vec{A} = \frac{\mu_0 I}{2\pi}\hat{y}\int_0^{y_{MAX}}\frac{dy}{\sqrt{y^2+z^2}} = \cancel{\frac{\mu_0 I}{2\pi}\hat{y}\left[\log z\right.}$$

$$\vec{A} = \frac{\mu_0 I}{2\pi}\hat{y}\left[\log\left|y_{MAX}+\sqrt{y_{MAX}^2+z^2}\right| - \log z\right]$$

This is the interesting part

Now let $y_{MAX} \to \infty$

$$\vec{A} = -\frac{\mu_0 I \log z}{2\pi}\hat{y} + \underbrace{\frac{\mu_0 I}{2\pi}\log 2y_{MAX}\,\hat{y}}_{\text{this is a constant}}$$

Since eventually $\vec{B} = \vec{\nabla}\times\vec{A}$, ie, I need to take a bunch of derivatives, the constant doesn't matter and can be dropped - Take $A = -\frac{\mu_0 I \log z}{2\pi}\hat{y}$

$$\vec{B} = \vec{\nabla}\times\vec{A} = -\frac{\mu_0 I}{2\pi}\begin{vmatrix}\hat{x} & \hat{y} & \hat{z}\\ \partial/\partial x & \partial/\partial y & \partial/\partial z\\ 0 & \log z & 0\end{vmatrix} = -\frac{\mu_0 I}{2\pi}\left(-\frac{1}{z}\right)\hat{x} = \frac{\mu_0 I}{2\pi z}\hat{x}$$